Dual-Modality Imaging

Davide Poggiali

November 21, 2013

Calendar of seminars

Sept. 19 **Physical Basis of Magnetic Resonance Imaging** ☑

spin under a magnetic field, RF pulses, Bloch equations

Sept.27 **Sequences and Reconstruction in MRI** ☑

K-space, aliasing, sequences

Oct. 18 **Positron Emission Tomography: an introduction** ☑

general introduction, attenuation correction, level set methods

Nov. 8 **Kinetics of the Tracer in PET** ☑

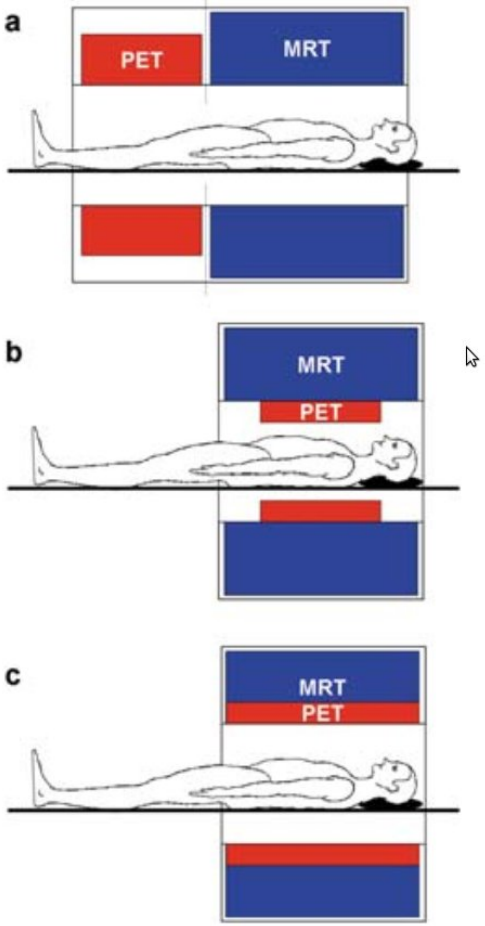
compartment model for ^{18}FdG

Nov. 21 **Dual-Modality Imaging** ☺

The challenges of PET/MR

PET/MRI is a new, arising hybrid-imaging technology that incorporates MRI morphological imaging and PET functional imaging.
At the moment 3 companies declare availability of their PET/MRI imaging devices:

- PHILIPS
- SIEMENS
- GE


a
PET
MRT
b
MRT
PET
c
MRT
PET

Consider the linear system $Ax = b$, with b projection (PET) data and x the image to reconstruct Maximum Likelihood Expectation Maximization (MLEM) is based on a probabilistic argument (the noise is assumed to be Poissonian).

$$L(x) = P(b|x) = \prod_{i=1}^{lp} \frac{(b_i^*)^{b_i}}{b_i!} e^{-b_i^*}$$

where $b^* = Ax$ is the exact sinogram, i.e. the projection of the exact solution x. Equivalently, it maximizes

$$l(x) = \log(L(x)) = \sum_{i=1}^{lp} -(Ax)_i + b_i \log(Ax)_i$$

Kuhn-Tucker conditions are then:

$$x_j \frac{\partial l(x)}{\partial x_j} = 0 \text{ if } x_j > 0$$

$$\frac{\partial l(x)}{\partial x_j} \leq 0 \text{ if } x_j = 0$$

From the first equation (the second is always satisfied) we get

$$0 = x_j \frac{\partial l(x)}{\partial x_j} = x_j \left(- \sum_{i'=1}^{lp} a_{i',j} + \sum_{i=1}^{lp} \frac{a_{i,j} b_i}{\sum_{j'=1}^{N^2} a_{i,j'} x_{j'}} \right)$$

this leads to the iterative scheme:

$$x_j \leftarrow \frac{x_j}{\sum_{i'=1}^{lp} a_{i',j}} \sum_{i=1}^{lp} \frac{a_{i,j} b_i}{\sum_{j'=1}^{N^2} a_{i,j'} x_{j'}}$$

If we write the scheme in a vectorial and multi-step form

1. $b^f := Ax^{(k)}$
2. $b^q := b./b^f$ (punctual division)
3. $x^b = A^T b^q$
4. $s_j = \sum_i a_{i,j}$
5. $x^{(k+1)} = x^{(k)}. * x^b./s$ (product and division are made elementwise)

The goal of a reconstruction iterative scheme is to minimize the distance $D(b, Ax)$. For instance the least-squares method proposes to minimize the euclidean distance

$$D_1(b, Ax) = \sum_i \left(\sum_j a_{i,j} x_j - b_j \right)^2 .$$

Note that using this distance leads to the normal equations.

The Bayesian methods are similar, but they add an extra penalty term D_2 to the objective function

$$J_\beta(x) = D_1(b, Ax) + \beta D_2(x)$$

with the constant $\beta \geq 0$ set up by the user.
Sometimes D_2 can be the distance between the current image and a prior image model p. In this case the objective function is

$$J_\beta(x) = D_1(b, Ax) + \beta D_2(x, p).$$

The cross-entropy or Kullback-Leiber distance is defined as it follows

$$S(u, v) = \sum_i u_i \log(u_i/v_i) - u_i + v_i.$$

Note that minimizing $S(b, Ax)$ is equivalent to minimizing the log-likelihood function $l(x)$ described above.
So we minimize

$$J_\beta(x) = S(b, Ax) + \beta S(x, p).$$

Minimizing this function leads to the iterative scheme:

$$x_j \leftarrow \frac{x_j}{\sum_{i'} a_{i',j}} \left[\sum_i \frac{a_{i,j} b_i}{\sum_{j'} a_{i,j'} x_{j'}} - \beta \log(x_j/p_j) \right]$$

This scheme is called MXE1 and converges provided the non-negativity Kuhn-Tucker conditions

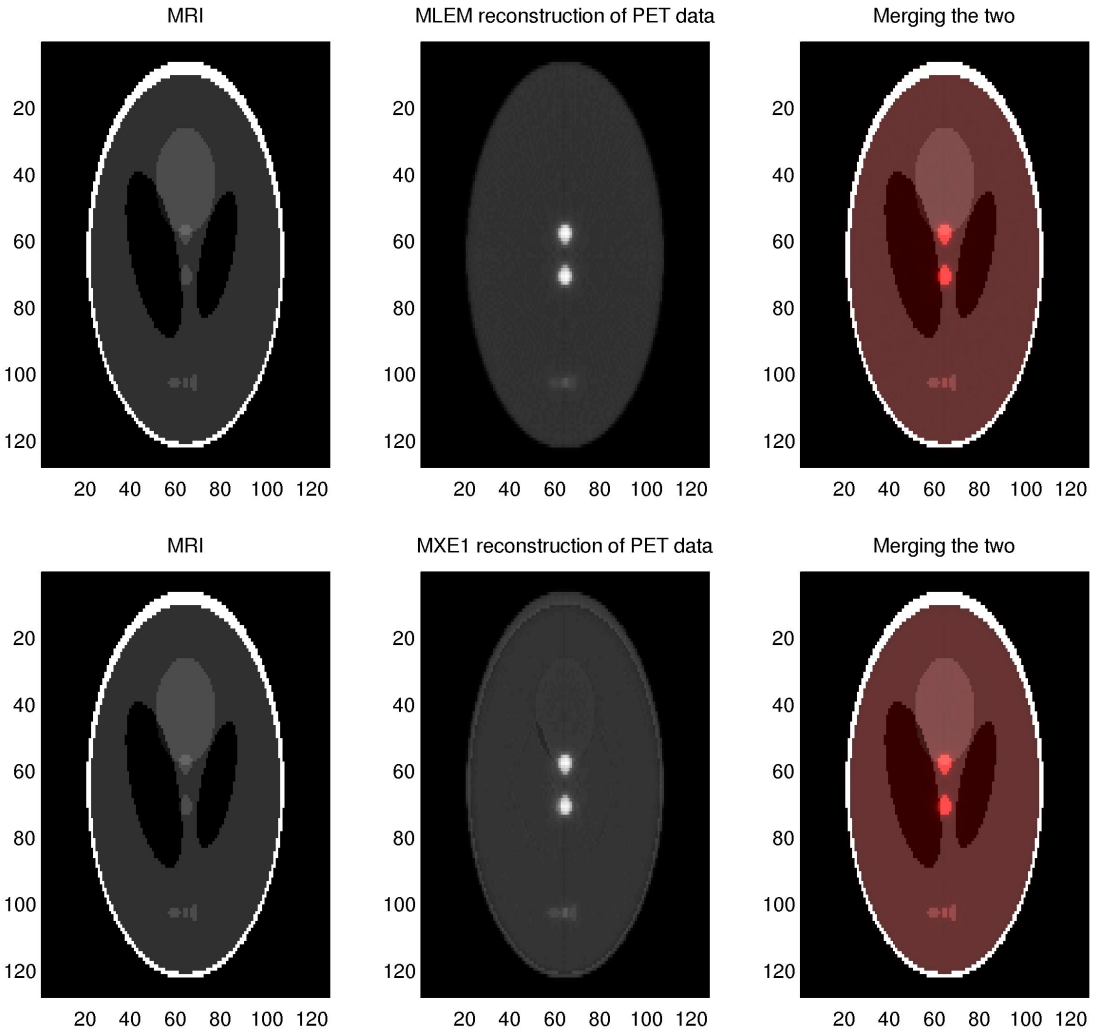
$$\sum_i \frac{a_{i,j} b_i}{\sum_{j'} a_{i,j'} x_{j'}} > \beta \log(x_j/p_j)$$

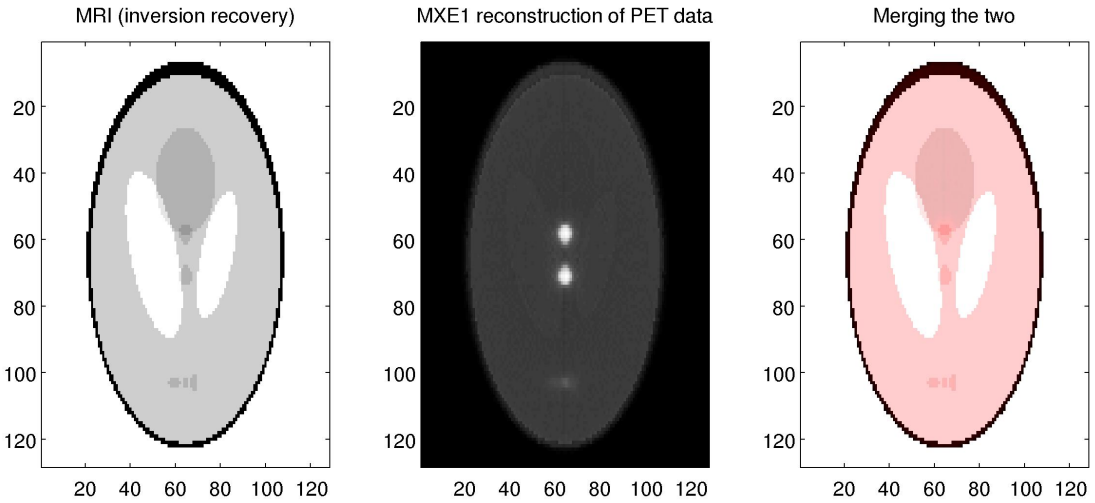
so for β too large this condition is violated.

Another iterative scheme for solving the optimization problem is

$$x_j \leftarrow p_j \exp\left[-\frac{1}{\beta}\left(\sum_i a_{i,j} - \sum_i \frac{a_{i,j} b_j}{\sum_{j'} a_{i,j'} x_{j'}}\right)\right]$$

this scheme is called MXE2. MXE2 satisfies the non-negativity constraint for all $\beta > 0$.


MRI
MLEM reconstruction of PET data
Merging the two
MRI
MXE1 reconstruction of PET data
Merging the two
20
40
60
80
100
120


MRI (inversion recovery)
MXE1 reconstruction of PET data
Merging the two
20
40
60
80
100
120

Essential bibliography

- Ardekani B.A., Braun M., Hutton B.F., Kanno I. and Iida I., *Minimum cross-entropy reconstruction of PET images using prior anatomical information*, Phys. Med. Biol. 41, 1996
- Catana C., Drzezga A., Heiss W.-D., Rosen B.R. ,*PET/MRI for Neurologic Applications*,J Nucl Med 2012
- Hofmann M., Pichler B., Schlkopf B., Beyer T.*Towards quantitative PET/MRI: a review of MR-based attenuation correction techniques*, Eur J Nucl Med Mol Imaging, 2009
- Schulz V., Torres-Espallardo I., Krombach G. A. et al. *Automatic, three-segment, MR-based attenuation correction for whole-body PET/MR data*, Eur J Nucl Med Mol Imaging 2011