Resolution of a Gamma Camera: experimental data and analytical formula

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June 4, 2013

Calendar of seminars

May 14 Introduction to Analytic Tomography ☑

general introduction, Radon tranform, FBP

May 21 Reconstruction in X-ray computed tomography ☑

iterative methods, kernel methods

May 28 The mathematics behind SPECT/TC reconstruction ☑

analytical and iterative hybrid reconstruction

June 4 Resolution of a Gamma Camera: experimental data and analytical formula ☐

resolution formulas

June 11 An alternative Radon transform for the correction of partial volume effect

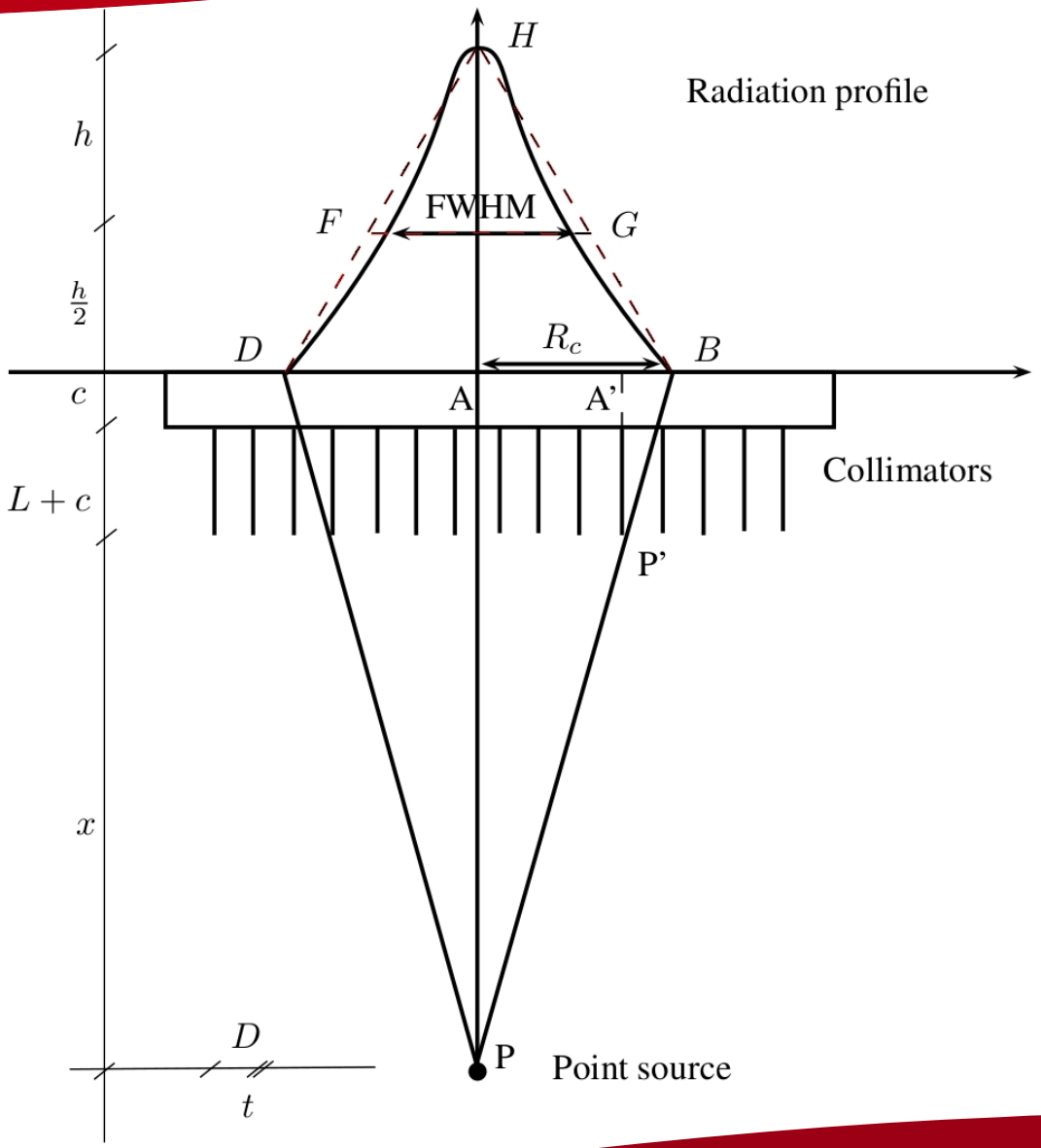
a modest proposal..

Analytical Resolution

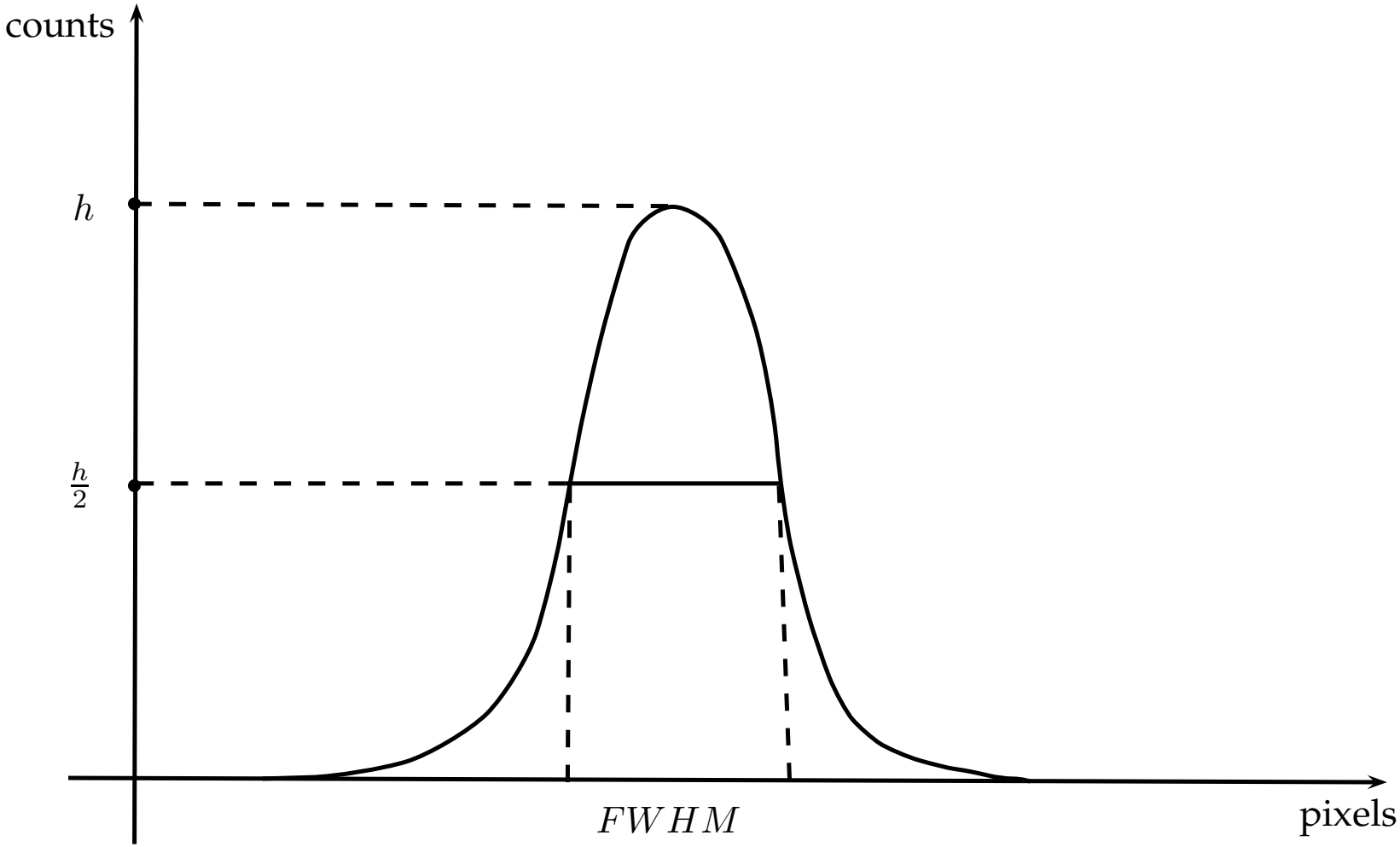
$$R_c = D\left(1 + \frac{x + c}{L_{eff}}\right)$$

The resolution of the machine depends on the collimator resolution and on the intrinsic resolution (the resolution of the crystal and the electronics).

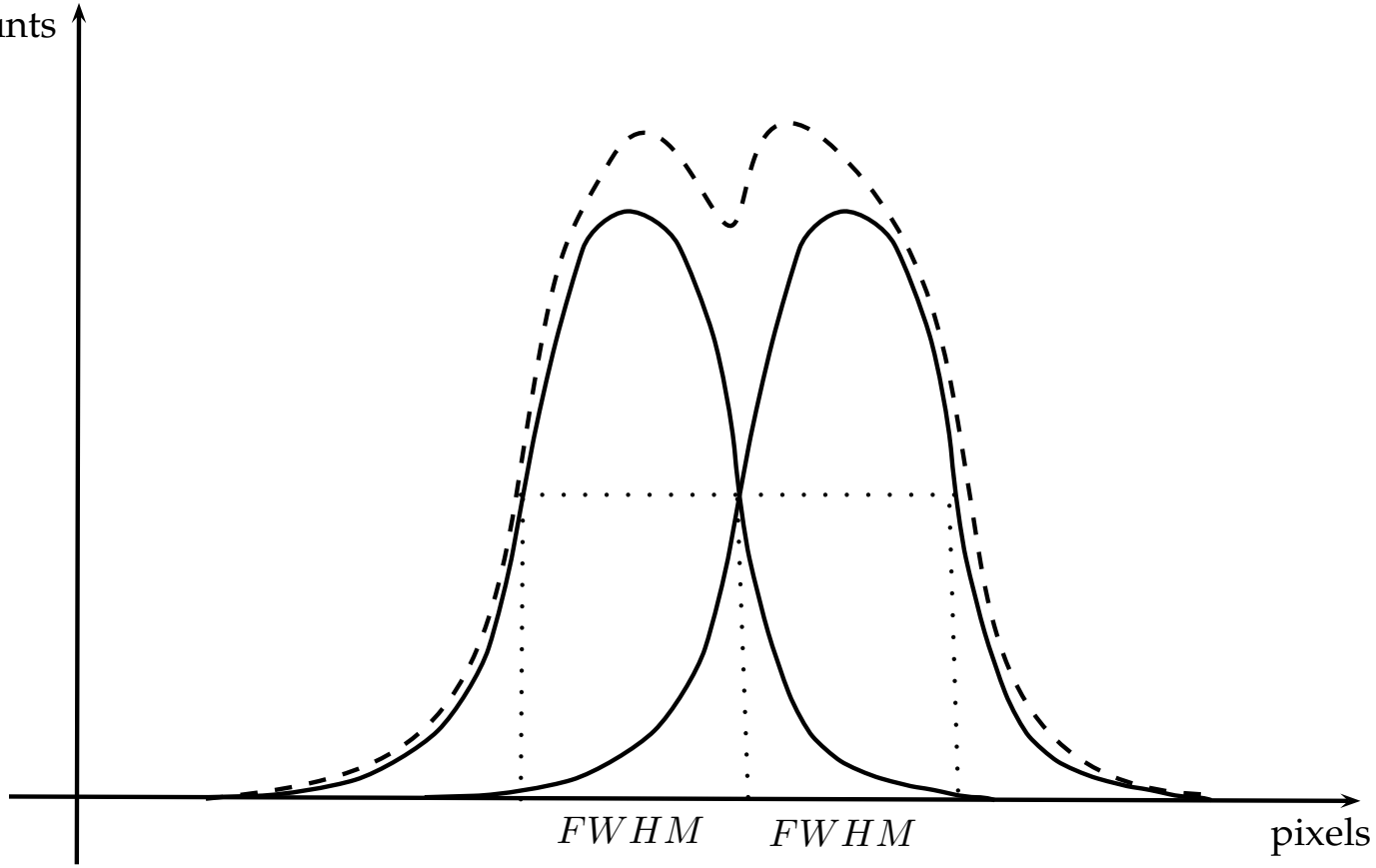
$$R_s = \sqrt{R_c^2 + R_i^2}$$


H
Radiation profile
h
F
FWHM
G
$\frac{h}{2}$
D
R_c
B
c
A
A'
Collimators
$L + c$
P'
x
D
t
P
Point source

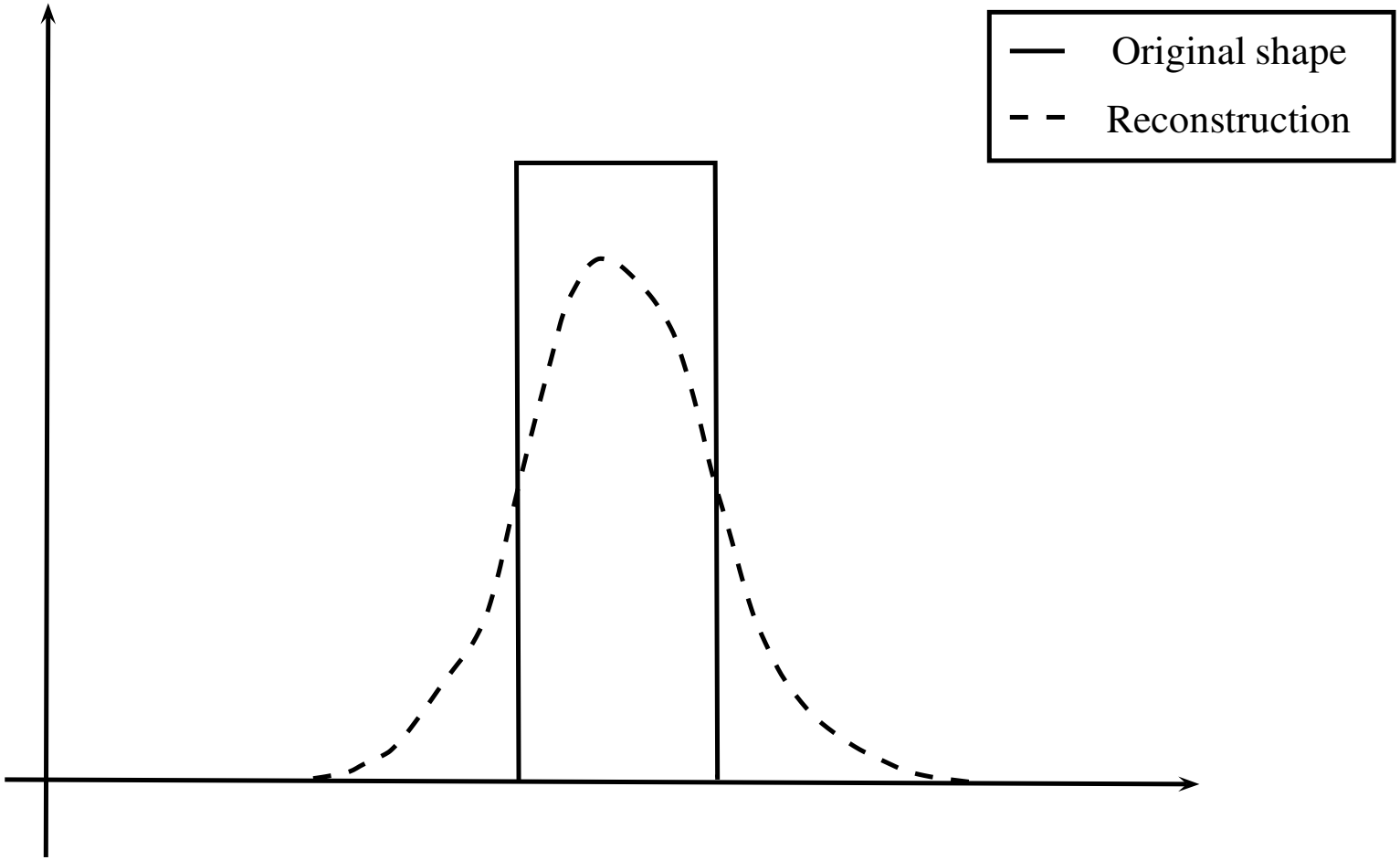
FWHM



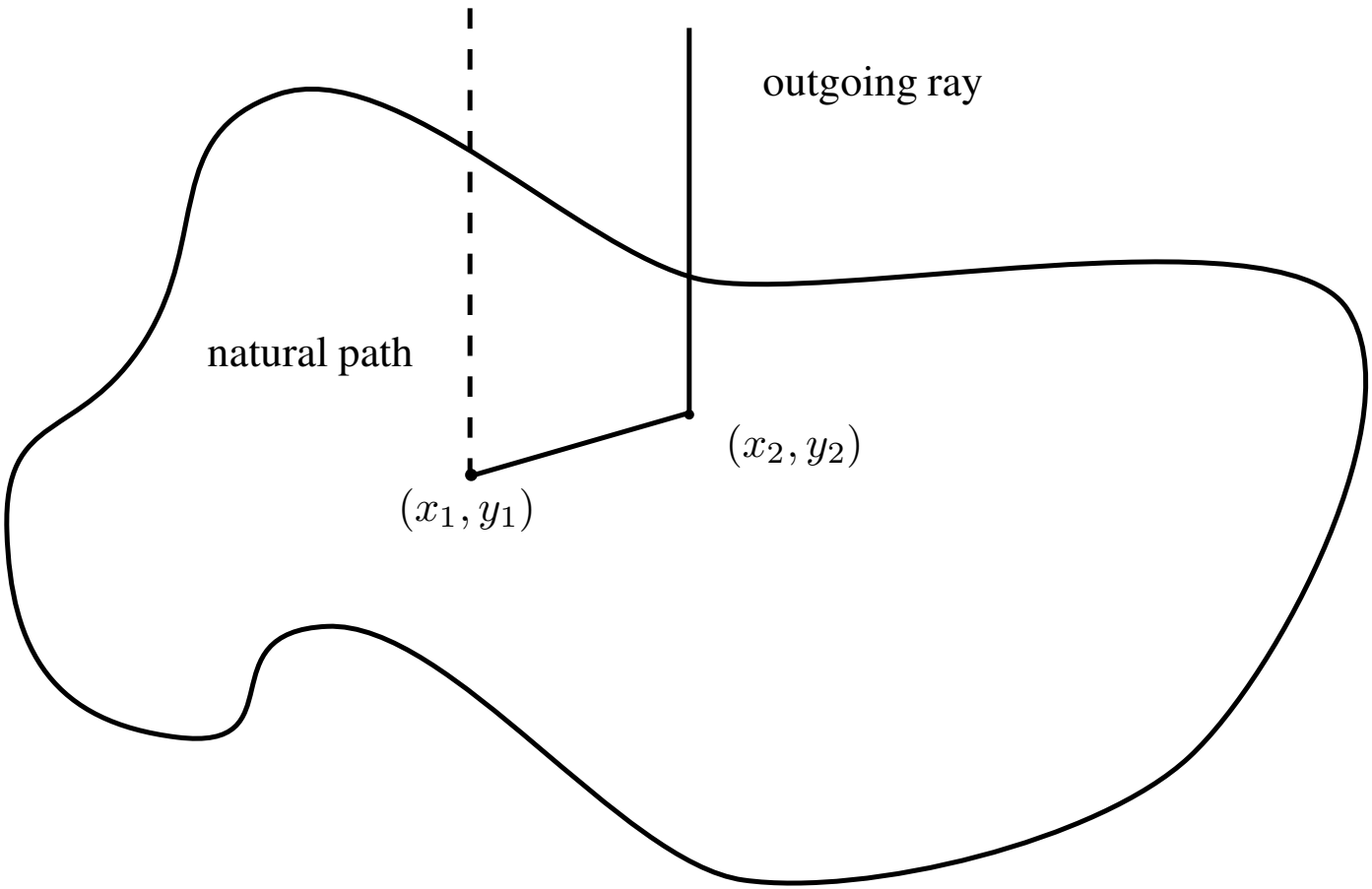
FWHM

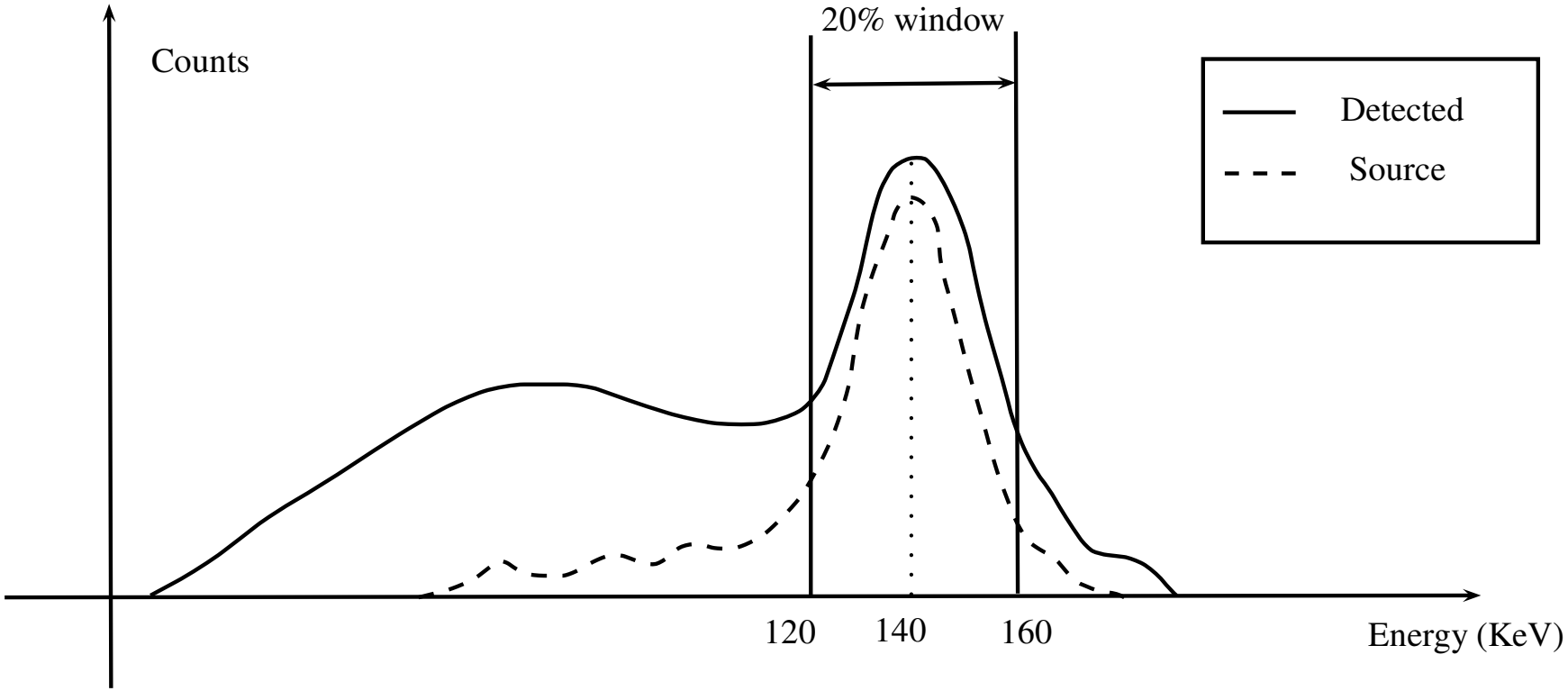


Partial volume effect

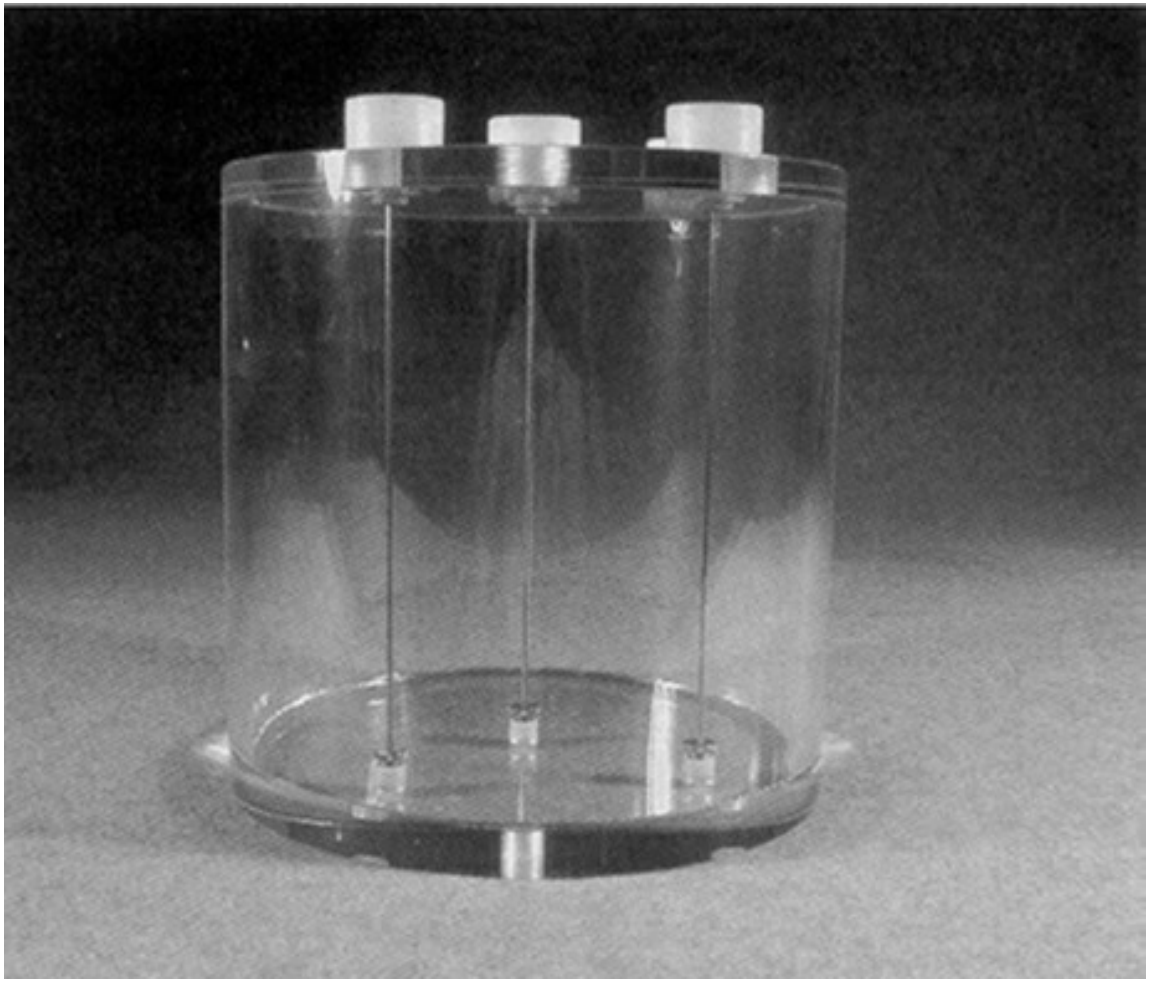


Scattering




Counts
20% window
Detected
Source
120
140
160
Energy (KeV)

The 3-capillaries phantom



We use four different methods for the filling of the phantom:

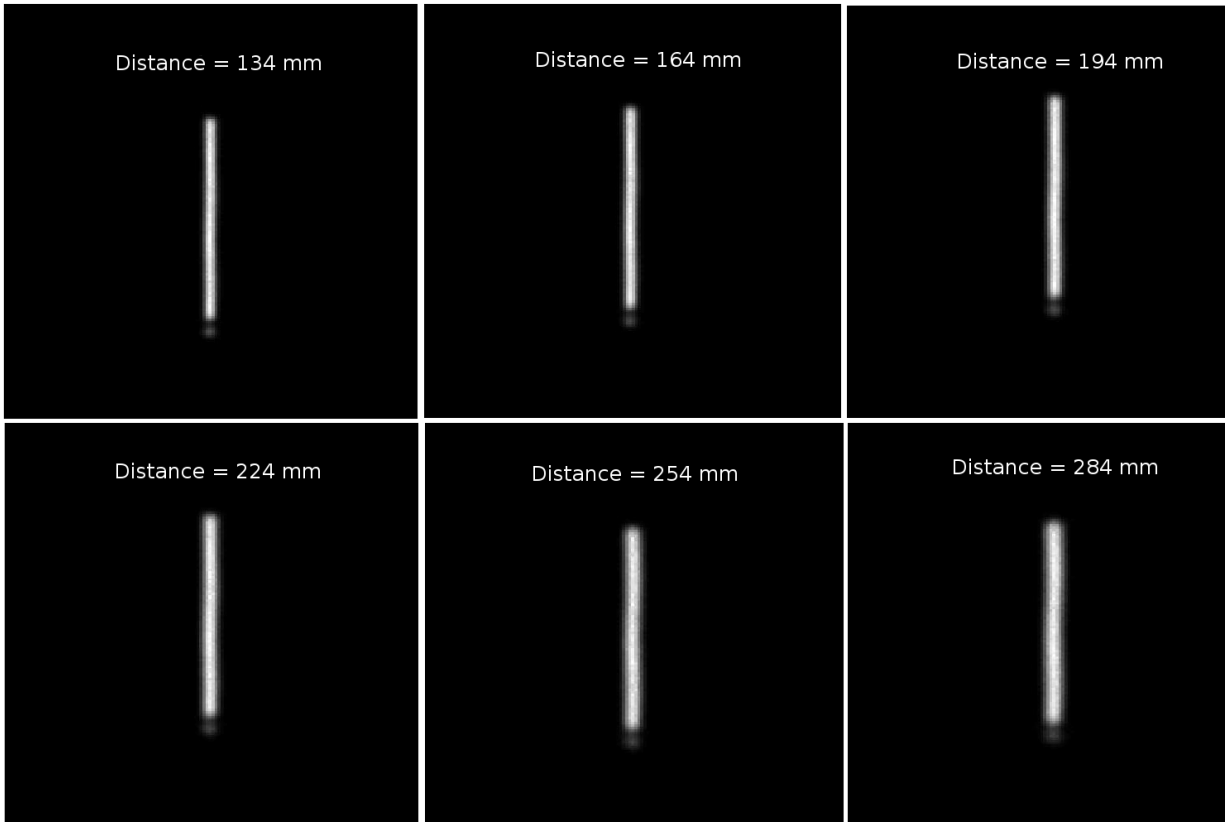
1. only one capillar filled with tracer is scanned (we call this experiment *in air*).
2. the central capillar filled with tracer and the rest filled with air (*in plexiglas*).
3. the central capillar filled with tracer and the rest filled with water (*in aqua*).

We have the parameters of the machine:

Descript.	L (mm)	D (mm)	c (mm)	t (mm)	FWHM @ 0mm	FWHM @ 100mm
UHR Irix	58.4	1.78	19	0.152	4.8 mm	6.7 mm

so we can find the analytical resolution $R_s(x) = \sqrt{\alpha x^2 + \beta x + \gamma}$.
From the known formula we get

$$\alpha = 0.0010, \quad \beta = 0.1468, \quad \gamma = 2.4267.$$



A graphic view on the data coming from the 3cap-phantom, an *in water* scan with the central capillary posed at several distances from the collimators. The color refers to the value of the data in the relative pixel.

We can use as mean value of FWHM, the mean value of FWHMs at J different heights

$$mean(FWHM) = \sum_{j \in J} \frac{FWHM_j}{|J|}$$

and thus obtaining more accurate results with an estimation of the *absolute error* given by the *standard deviation* (or s.d.) of the results

$$\sigma = \sqrt{\frac{\sum_{j \in J}(FWHM_j - mean(FWHM))^2}{|J|}}$$

The first method is quite simple: let us consider the data at a given heigh j, we call them $(x_i, y_i)_{i=1,\dots,N}$.
We find the maximum for the second value $h = \max(y_i)$ and the relative argument $\tilde{x}$. We take the two points $z_1 < \tilde{x}$ and $z_2 > \tilde{x}$ which are closer to the half of the maximum and calculate their distance

$$FWHM = |z_1 - z_2|.$$

For this case we have used the following cost

$$C_1(y_1, y_2) = \frac{(y(z_1) - h/2)^2 + (y(z_2) - h/2)^2}{2}$$

This method is very fast, but also very inaccurate.

Now we think our data as they come from a deterministic function $y = f_{\bar{a}}(x)$ plus a small level of noise. Let $(x_i, y_i)_{i=1,\dots,N}$ be the experimental data at a given height, then we look for

$$\bar{a}^* = \arg\min_{\bar{a}\in\mathbb{R}^n} J(\bar{a}) = \arg\min_{\bar{a}\in\mathbb{R}^n} \|y_i - f_{\bar{a}}(x_i)\|^2 .$$

Since not always f_a depends linearly from the parameters $\bar{a}$ we use an iterative optimization algorithm to estimate $\bar{a}^*$.

The cost used in with this method is

$$C_2(\bar{a}) = \frac{J(\bar{a}^*)}{N}$$

We choose three different functions $f_{\bar{a}}$.

(i) The *gaussian*

$$f_{\bar{a}}(x) = a_1 e^{-a_2^2 x^2}$$

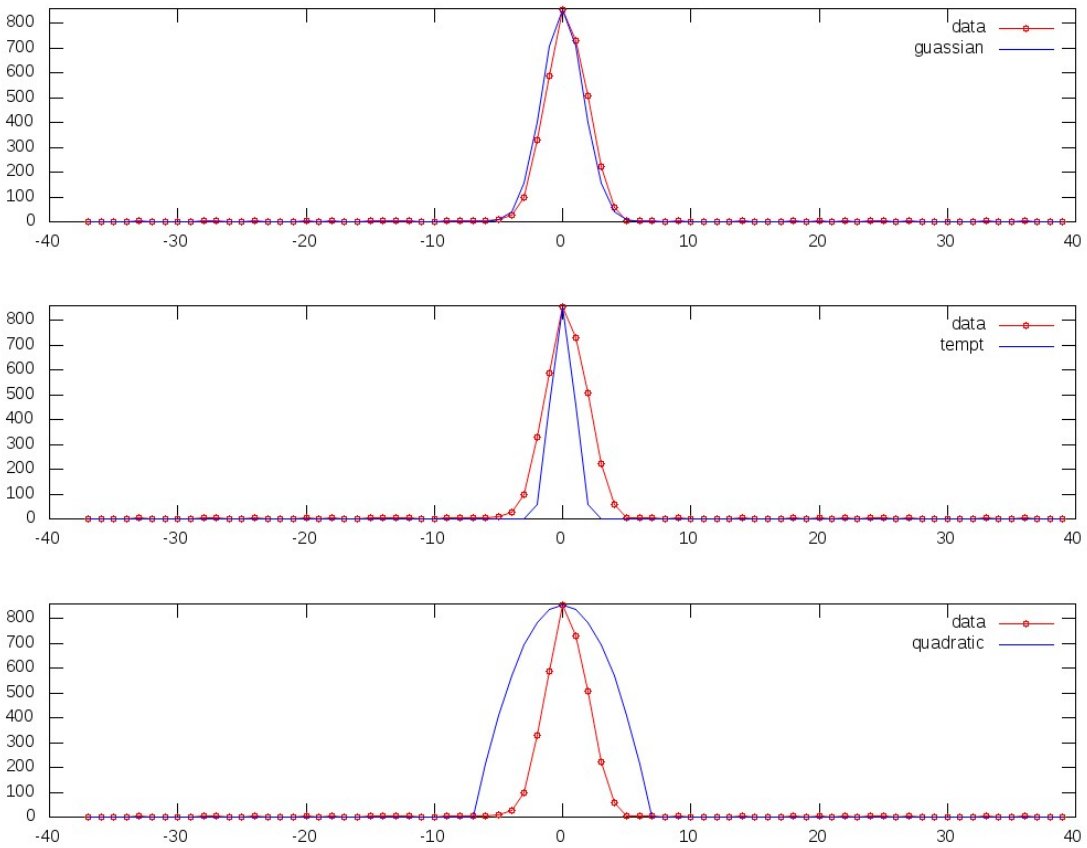
(ii) The *tempt*

$$f_{\bar{a}}(x) = \max\left((-a_1 x + a_2)\chi_{x>0} + (a_1 x + a_2)\chi_{x\leq 0}, 0\right)$$

w

(iii) The *truncated quadratic*

$$f_{\bar{a}}(x) = (a_1 x^2 + a_2)_+ = \max(a_1 x^2 + a_2, 0) \quad \text{with } a_1 < 0\,,$$



The Gaussian function is the best one.

Now we want to approximate the data $(x_i, y_i)_{i=1,\dots,N}$ by a function s defined as follows.

1. On each subinterval $I_i = [x_i, x_{i+1}],\ i = 1, \dots, N-1$, $s_{|I_i} = s_i \in \mathcal{P}^m(\mathbb{R})$ where $\mathcal{P}^m(\mathbb{R})$ is the space of real polynomials of degree $\leq m$.
2. $s(x_i) = y_i\ \forall i = 1, \dots, N$, that is f interpolates the data.
3. $s_i^{(k)}(x_i) = s_{i+1}^{(k)}(x_i),\ \ \forall i = 2, \dots, N-1,\ k \leq m-1,\ m \leq N$. This means that the polynomial pieces are countinuous up to order $m-1$ in each interior points.

This approach is called *local interpolation* and the functions f are known as (polynomial) *splines* of order m.

As in method 1, we have found two points X_1 and X_2 whose distance from the half of the maximum is minimal. To be more precise we find

$$z_i = \arg\min_{x \in I_i} |s(x) - h/2|$$

with $I_1 = \mathrm{linspace}(x_1, \tilde{x}, 10^4)$ and $I_2 = \mathrm{linspace}(\tilde{x}, x_N, 10^4)$, $h = max(y_i)$ and $\tilde{x}$ the argument of the previous maximum, as in Method 1.

The distance of these two points will be a precise estimate of the FWHM

$$FWHM = |z_1 - z_2|.$$

The cost is defined as in method 1

$$C_3(z_1, z_2) = \frac{(s(z_1) - h/2)^2 + (s(z_2) - h/2)^2}{2}$$

From a comparison made on the cost the third method results the most accurate one.

Method	Direct	Gloabal interp	Local interp
Mean cost	$1.5 \cdot 10^3$	0.21	0.15
Mean v.c.	$9,3\%$	2.4%	3.0%

Since the collimator resolution is linear with the distance and the Intrinsic Resolution R_i is constant, then the dependence from the distance of the System Resolution can be modelled by a function

$$R_s(x) = \sqrt{\alpha x^2 + \beta x + \gamma}.$$

Now we interpolated the FWHM experimental data using such model and then we compared the fitting curve with the "analytical" one.

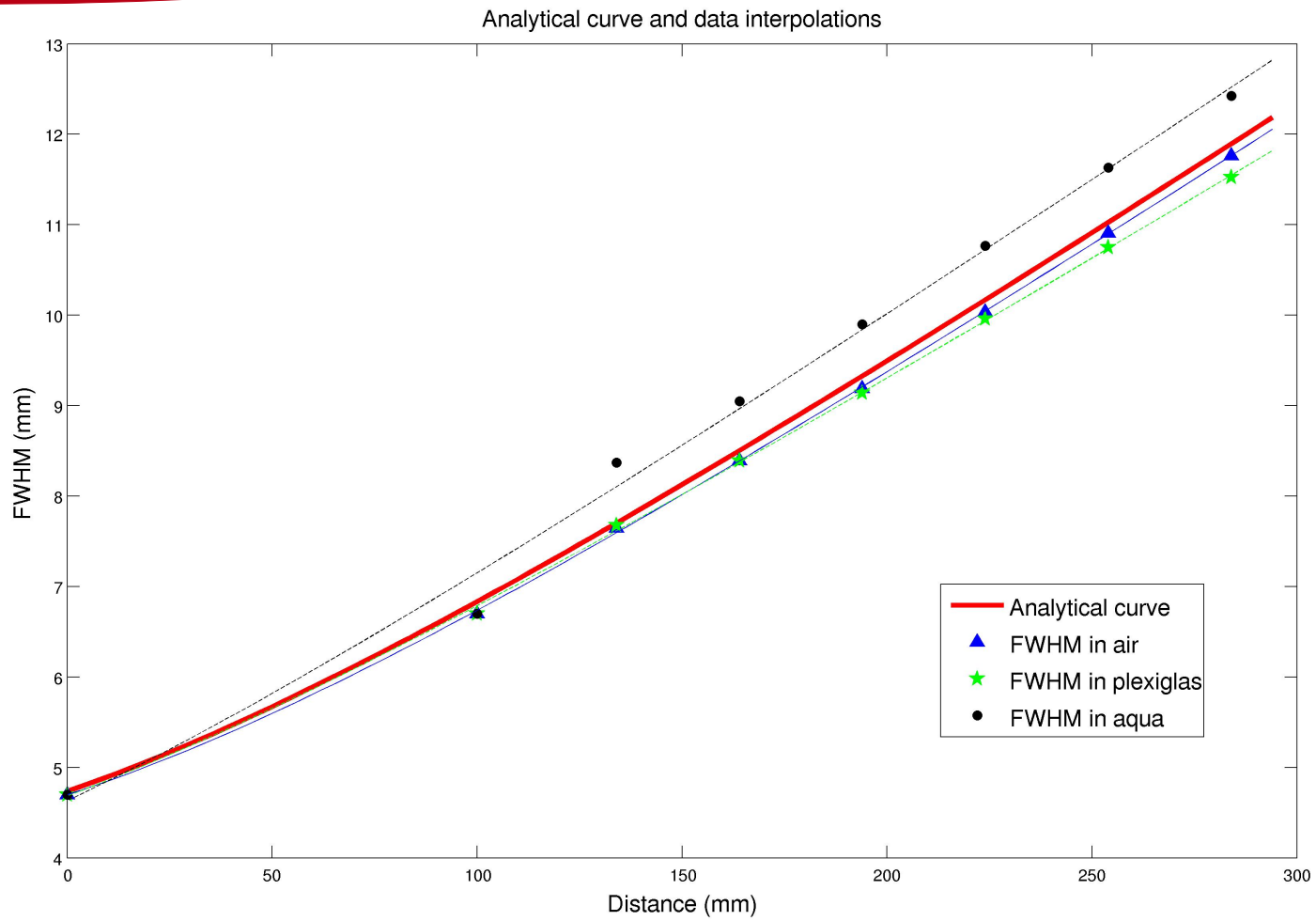
The fitting curve as been computed as a *weighted least-squares method* (cf. [2]). We considered

$$y^2 = ax^2 + bx + c\,,$$

with y the vector of FWHMs, the vector $(y^2)_j = (y_j^2)$ and x the vector of distances, so that the system is linear on its parameters. We found the parameters $p = (a, b, c)$ by solving the normal equations

$$(V^T \Sigma V)p = V^T \Sigma f$$

where V is the Vandermonde matrix, $f = y^2$ and $\Sigma = diag(1/\sigma_1^2, \ldots, 1/\sigma_n^2)$ the weight matrix, with σ_i the standard deviation (in mm) of the i-th resolution value.



Analytical and experimental resolution with different levels of scattering.

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