An alternative Radon transform for the correction of partial volume effect

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Calendar of seminars

May 14 Introduction to Analytic Tomography ☑

general introduction, Radon tranform, FBP

May 21 Reconstruction in X-ray computed tomography ☑

iterative methods, kernel methods

May 28 The mathematics behind SPECT/TC reconstruction ☑

analytical and iterative hybrid reconstruction

June 4 Resolution of a Gamma Camera: experimental data and analytical formula ☑

resolution formulas

June 28 An alternative Radon transform for the correction of partial volume effect ☐

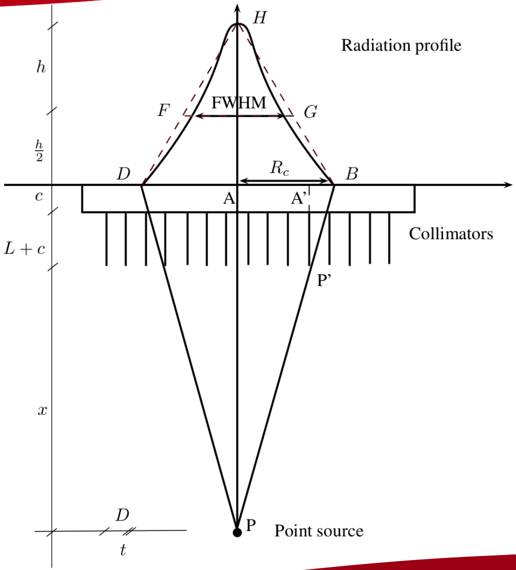
a modest proposal..

Analytical Resolution

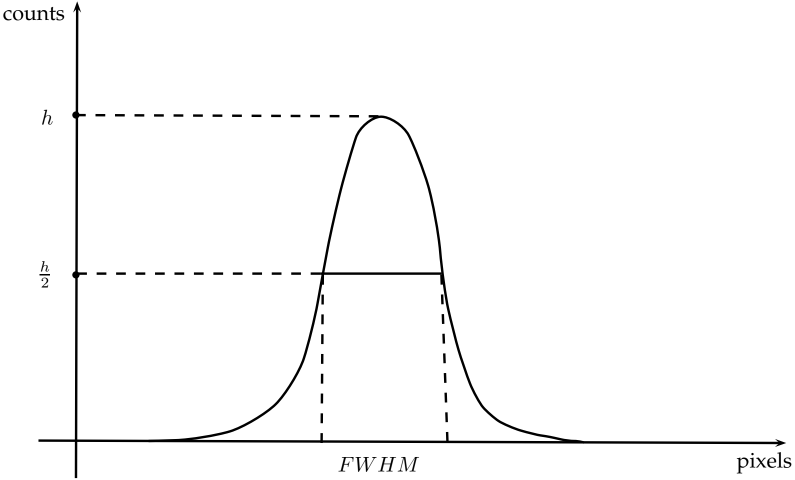
$$R_c = D\left(1 + \frac{x+c}{L_{eff}}\right)$$

The resolution of the machine depends on the collimator resolution and on the intrinsic resolution (the resolution of the crystal and the electronics).

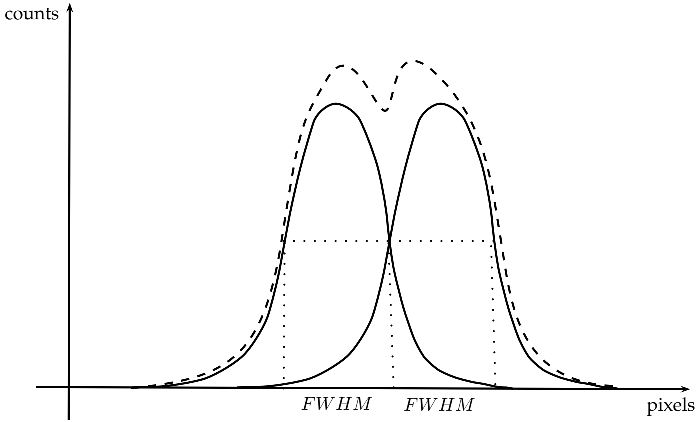
$$R_s = \sqrt{R_c^2 + R_i^2}$$


H
Radiation profile
h
F
FWHM
G
$\frac{h}{2}$
D
R_c
B
c
A
A'
$L+c$
Collimators
P'
x
D
P
Point source
t

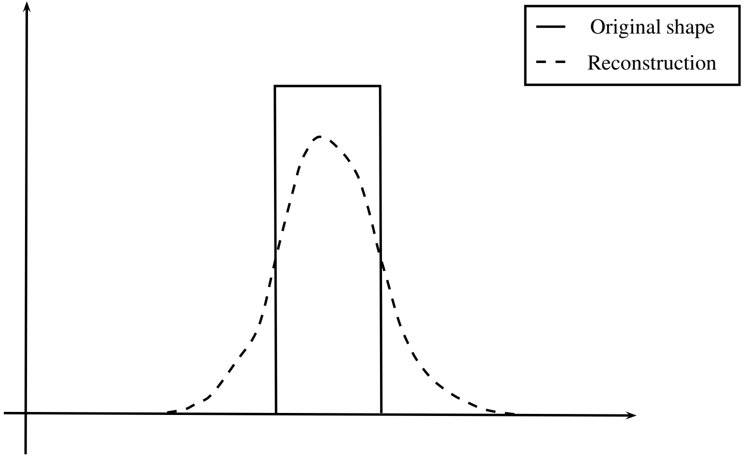
FWHM



FWHM

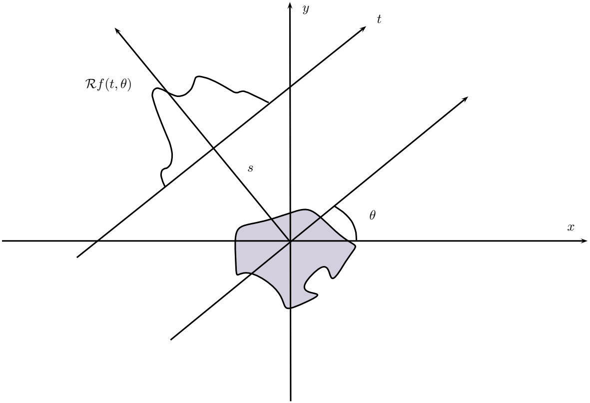


Partial volume effect



In order to model correctly the tomography machine we will make some assumptions:

- all rays have no width and we can consider them as straight lines.
- all rays at angle θ are parallel.
- the machine runs I scans at the angles $(\theta_1, \ldots, \theta_I)$; the detectors move on a circle (typically $I = 120, 60$).
- for each scan $j \in 1, \ldots, I$ the machine gets p linear samples (typically $p = N$, or $p = 128$ or $p = 64$).
- the total time of acquisition is relatively short so is indifferent to run an acquisition before or after another one.
- the reconstructed image has dimension $N \times N$ (typically $N = 128$ or $N = 64$).



$$\mathcal{R}f(t,\theta) = \int_{\mathbb{R}} f(t\cos\theta - s\sin\theta, t\sin\theta + s\cos\theta)\, ds$$

Now we note that this model is also based on another assumption: the ray is a whole straight line, so the collimator “sees” all the rays in his line, also the rays behind it.
This is not the only unreal hypothesis of the Radon transform: in fact not only the parallel rays will enter, because no collimator can have a resolution equal to zero!

Now the resolution is defined as the FWHM of the PSRF (Point Source Response Function), which is the data collected by the machine using a point source as input.
The machine sees every image I in input as

$$I = I_0 * PSRF$$

Where I_0 is the exact map and I is the map as seen by the machine. By '$*$' of course we mean the 2-D convolution product.

Now we make the (realistic) hypothesis that the PRSF function is a Gaussian

$$PRSF(x) = \exp(-\varepsilon^2 x^2)$$

Since we know that the FWHM of such function is $FWHM = \frac{2\sqrt{\ln 2}}{\varepsilon} = R(d)$ where d is the distance between source and collimator and $R = R_s$.

$$PRSF(x) = \exp\left(-\frac{4\ln 2}{R^2(d)}x^2\right)$$

if we put this expression into the new transform we gets

$$\mathfrak{R}f(t,\theta) = \iint_{[-r,r]^2} \exp\left(-\frac{4\ln 2\tau^2}{R^2(r-s)}\right) f((t-\tau)\bar{\theta} + s\bar{\theta}^{\perp})\, ds\, d\tau.$$

Since τ is the distance between the parallel ray and the other parallel rays we are considering and the distance source-collimator is $r-s$. The expression for R^2 is simply $R(d) = ad^2 + bd + c$.

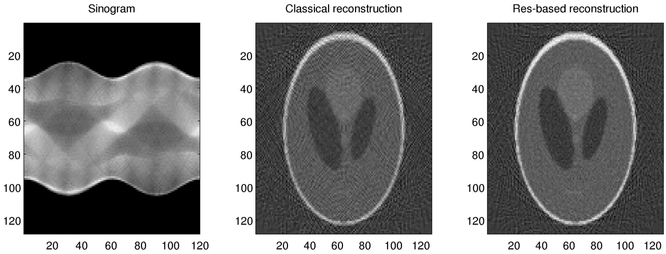
Since the integral is not easy to calculate for $f = \chi_{P_i}$, we calculate the matrix using the following approximation

$$R_{i,j} = \sum_{j' \in [j-\lambda, j+\lambda]} \exp\left(-\frac{4\ln 2(d(j-j')^2)}{R^2(D_{j,j'})}\right) A_{i,j'}$$

where A is the (classical) natural pixel matrix, d is the distance between a ray and the next ($d = \frac{N\sqrt{2}}{p}$), λ is an integer such that

$$\exp\left(-\frac{4\ln 2(\lambda^2)}{R^2(D_{j,j'})}\right) \ll 1$$

and D is a matrix that contains the distances between the $j' - th$ pixel and the collimator when the angle is θ_j.



The reconstruction using the matrix R is more accurate because the transformation has been done using R.

In this figure we can observe that the reconstruction using the matrix R is more accurate, since the partial volume effect seems to be corrected.

Pros:

- The new method is as fast as the classical one
- From the first experiments it seems to do what we asked
- The projection of a phantom looks like a sinogram.

Cons:

- New matrix as sparse as the classical one
- There is not a strong theory that supports our choices
- This kind of approach is completely work-in-progress.

Essential bibliography

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- Hansen P.C., Hansen M., *AIR Tools - A MATLAB Package of Algebraic Iterative Reconstruction Methods*, Journal of Computational and Applied Mathematics, 2011
- Natterer F., *The mathematics of computerized tomography*, SIAM: Society for Industrial and Applied Mathematic, 2001
- Toft P., *The Radon transform; theory and implementation*, Ph.D. thesis. Department of Mathematical Modeling, Technical University of Denmark, 1996
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