Sequences and Reconstruction in MRI

Davide Poggiali

September 26, 2013

Calendar of seminars

Sept. 19 Physical Basis of Magnetic Resonance Imaging ☑

spin under a magnetic field, RF pulses, Bloch equations

Sept.26 Sequences and Reconstruction in MRI ☐

K-space, aliasing, sequences

Oct. 10 Positron Emission Tomography: an introduction ☐

general introduction, attenuation correction, level set methods

Oct. 17 Kinetics of the Tracer in PET ☐

compartment model for ^{18}FdG

Oct. 31 Dual-Modality Imaging ☐

First Bloch equation (only static field)

$$\dot{\mathbf{M}} = \gamma \mathbf{M} \times \mathbf{B} - \left(\frac{M_x}{T_2}, \frac{M_y}{T2}, \frac{M_z - M_{eq}}{T_1} \right) \qquad \text{(Bloch-1)}$$

The solution to this ODE is

$$\begin{cases} M_x &= e^{-t/T_2}(M_{x,0} \cos \omega_0 t - M_{y,0} \sin \omega_0 t) \\ M_y &= e^{-t/T_2}(M_{x,0} \sin \omega_0 t + M_{y,0} \cos \omega_0 t) \\ M_z &= M_{z,0}\, e^{-t/T_1} + M_{eq}(1 - e^{-t/T_1}) \end{cases} \qquad (1)$$

Second Bloch equation (static field and RF pulse, in rotating reference)

$$\begin{cases} \dot{u} & = & 0 \\ \dot{v} & = & \gamma B_1 M_z \\ \dot{M}_z & = & -\gamma B_1 v \end{cases} \tag{Bloch-2}$$

The solution of this equation is simple

$$\begin{cases} u & = & u_0 \\ v & = & v_0 \cos \omega_1 t - M_{z,0} \sin \omega_1 t \\ M_z & = & v_0 \sin \omega_1 t + M_{z,0} \cos \omega_1 t \end{cases} \tag{2}$$

Third Bloch equation (static field and gradient, in rotating reference)

$$\begin{cases} \dot{u} & = & \gamma B_g v \\ \dot{v} & = & -\gamma B_g v \\ \dot{M}_z & = & 0 \end{cases} \tag{Bloch-3}$$

the solution to this ODE is similar to the previous one

$$\begin{cases} u & = & u_0 \cos \omega_g t - v_0 \sin \omega_g t \\ v & = & u_0 \sin \omega_g t + v_0 \cos \omega_g t \\ M_z & = & M_{z,0} \end{cases} \tag{3}$$

An RF pulse has the effect to flip the vector **M** from the axis z of an angle $\theta = \omega_1 \tau$ after a time τ. Thus we can apply following pulses:

- a general θ-**pulse**: $\tau = \frac{\theta}{\omega_1}$
- a $\frac{\pi}{2}$-**pulse**: $\tau_1 = \frac{\pi}{2\omega_1}$.
  After this pulse the aggregate magnetic momentum is
  $$\mathbf{M}(\tau_1) = (u_0, M_{z,0}, v_0).$$
- a π-**pulse**: $\tau_2 = \frac{\pi}{\omega_1}$.
  After this pulse the aggregate magnetic momentum is
  $$\mathbf{M}(\tau_2) = (u_0, -v_0, -M_{z,0}).$$

We can play with pulses to get more informations about the precession. The following sequences are the most popular ones:

1. **inversion recovery**
2. **saturation recovery**
3. **spin-echo**

Inversion recovery: after the we apply a π-pulse, we wait a time τ, then we apply a $\frac{\pi}{2}$-pulse and we measure $\mathbf{M}$. We wait for the system to get back to the equilibrium of (1) and we repeat several times to increase the signal-to-noise ratio and for several values of τ. This sequence is shorted as follows

$$(\pi - \tau - \frac{\pi}{2} - AT - t_\infty)_n$$

where AT stands for acquisition time, and t_∞ is a long time needed to get back to the equilibrium, usually $t_\infty \approx 4T_1$.
From the third component of (1) we get

$$M_z(\tau) - M_{eq} = (M_{z,0} - M_{eq})\exp(-\frac{\tau}{T_1}).$$

This means that we can obtain $-1/T_1$ as the slöpe of $\log(M_z(\tau) - M_{eq})$.

Saturation recovery: This sequence is shorted as

$$(\frac{\pi}{2} - Hs - \tau - \frac{\pi}{2} - AT - Hs)_n$$

Where Hs is a pulse that destroys the homogeneity of the field B_0; after a time τ the magnetization is partially recovered, the pulse fplips it on the xy-plane. Again, we obtain $-1/T_1$ as the slöpe of $\log(M_z(\tau) - M_{eq})$.

Spin echo: This sequence is shorted as

$$(\frac{\pi}{2} - \tau - \pi)_n$$

After 2τ the amplitude of the transverse magnetization will be relaxed by a factor

$$\exp(2\tau/T_2)$$

T2* is the real time constant of loss in the xy-plane, because the field $\mathbf{B_0}$ is not really homogeneous. The relation between $T2$ and $T2^*$ is quite simple:

$$\frac{1}{T2^*} = \frac{1}{T2} + \gamma \triangle \mathbf{B_0}$$

where $\triangle \mathbf{B_0}$ is the maximum variance of $\mathbf{B_0}$.

The aim of reconstruction is to create a 2D map of the proton density $\rho(\mathbf{x})$ on each slice. For this purpose we consider only the transverse component of $\mathbf{M}$,

$$M^* = M_x + iM_y$$

and same thing for the signal received from the coils

$$s = S_x + iS_y$$

For Faraday's law the magnetic field generated by the body induces an electromotive force to the coils

$$\mathbf{S}(\mathbf{t}) = \frac{d}{dt}\Phi_{\text{loop}} = k\frac{d}{dt}\int_{\text{loop}} \rho(\mathbf{x})\mathbf{M}dx$$

Now, if we consider a tiny slice over the z-axis, we can rewrite the previous formula as

$$s(t) = k_1\frac{d}{dt}\int_{\text{loop}'} \rho(\mathbf{z})M^*(\mathbf{z})dz$$

with $\mathbf{z} \in \mathbb{C}^2$.

If we apply a $\frac{\pi}{2}$-pulse, then a gradient for the selection of a slice, we get

$$M^* = \exp\left(-\frac{t}{T_2} + i\omega_0 t + i\omega_g t\right)$$

then we obtain

$$s(t) = k_2 \exp\left(-\frac{t}{T_2} + i\omega_0 t\right) \int_{\text{loop}'} \rho(\mathbf{z}) \exp(i\omega_g t) d\mathbf{z}.$$

We remember that $\omega_g = -\gamma B_g = -\gamma \mathbf{G} \cdot \mathbf{x}$ then, if we set

$$G = G_x + iG_y$$

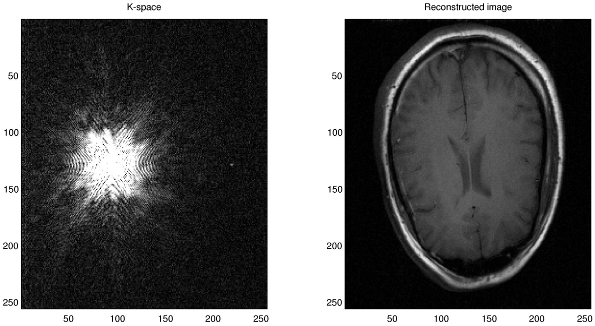
we get

$$s(t) = k_3 \exp\left(-\frac{t}{T_2} + i\omega_0 t\right) \int_{\text{loop}'} \rho(\mathbf{z}) \exp(-it\gamma G \cdot \mathbf{z}) d\mathbf{z}$$

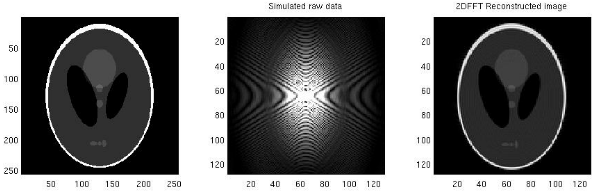
If we look at the previous formula, we have found that

$$s(t) \propto \mathcal{F}_2[\rho](k(t))$$

where of course $k(t) = -\frac{t\gamma}{2\pi} G$.
So we store the values of $\frac{s(t)}{k_3 \exp\left(-\frac{t}{T_2} + i\omega_0 t\right)}$ in a matrix called K-space and we apply the 2-D IFFT algorithm to recover the slice image.



An example of K-space and reconstructed image.



We can also use a phantom to obtain a simulated K-space.

MRI is a multi-parametric imaging system: in fact you can play with some parameters to obtain different contrast between the tissues.
We now introduce some of these parameters.

TR, Repetition Time is the time interval between the application of a pulse and another.
After TR the longitudinal magnetization of the vector **M** will be

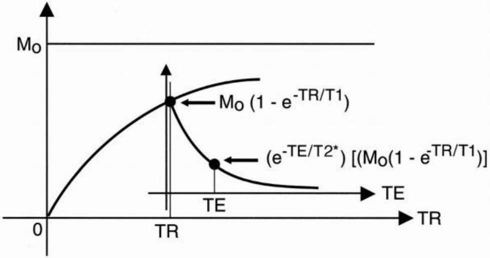
$$M_z(t) = M_{eq}\left(1 - \mathrm{e}^{-TR/T1}\right)$$

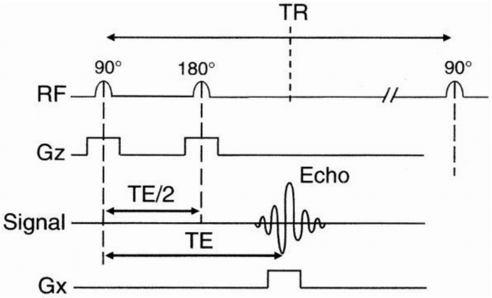
so each FID signal will be proportional to

$$\left(1 - \mathrm{e}^{-TR/T1}\right)$$

This means that the longitudinal magnetization will not reach M_{eq} unless we set $TR \gg 4T1$

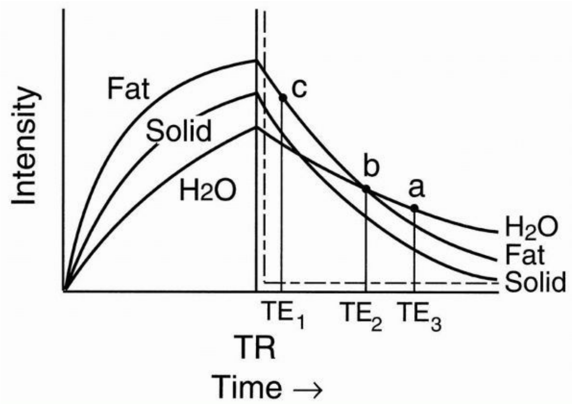
TE stands for echo delay time (or time to echo). Instead of making the measurement immediately after the RF pulse (impossible), we wait a short period of time and then make the measurement.

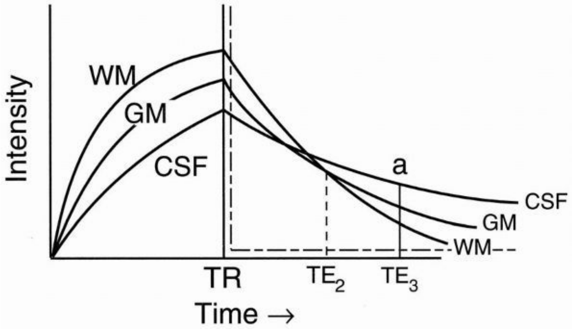




A complete example of spin-echo sequence.

- T1-weighted short TR
  Useful in scanning brain to see contrast between gray and white matter.
- T2-weighted long TE, long TR
  Used to see the contrast between water and fat.
- $T2^*$-weighted long TE, long TR
  We use it to see particulars in venous blood.
- Proton density weighted short TE, long TR
  We try not to consider the effect of $T1$ or $T2$.


Intensity
Fat
Solid
H2O
c
b
a
H2O
Fat
Solid
TE1
TE2
TE3
TR
Time →


Intensity
WM
GM
CSF
a
CSF
GM
WM
TR
TE$_2$
TE$_3$
Time →

Essential bibliography

- Callaghan P., *Introductory NMR & MRI*, a video course, http://www.youtube.com/watch?v=7aRKAXD4dAg
- Epstein C.L., *Introduction to the mathematics of Medical Imaging*, Second Edition, Siam, 2008, Ch14.
- Feeman T.G., *The mathematics of medical imaging: A beginners guide*, Springer, 2010, Ch10.
- Hashemi R.H., Bradley W.G. Jr, Lisanti C.J., *MRI, the Basics*, Lippincott Williams & Wilkins, 2010.